Problem A.3

Use Equation A.13 to simplify the expression $\delta(\sin x)$. Sketch this function.

Solution

Equation A.13 is on page 426.

$$\delta(g(x)) = \sum_{i=1}^{n} \frac{1}{|g'(x_i)|}\delta(x - x_i) \tag{A.13}$$

In this problem $g(x) = \sin x$. Find the zeros of this function.

$$\sin x = 0$$

$$x_i = i\pi, \quad i = 0, \pm 1, \pm 2, \ldots$$

Now evaluate the derivative of $g(x)$.

$$\begin{aligned} g'(x) &= \frac{d}{dx}(\sin x) \\ &= \cos x \end{aligned}$$

Therefore, by Equation A.13,

$$\begin{aligned} \delta(\sin x) &= \sum_{i=-\infty}^{\infty} \frac{1}{|g'(i\pi)|}\delta(x - i\pi) \\ &= \sum_{i=-\infty}^{\infty} \frac{1}{|\cos i\pi|}\delta(x - i\pi) \\ &= \sum_{i=-\infty}^{\infty} \frac{1}{|(-1)^i|}\delta(x - i\pi) \\ &= \sum_{i=-\infty}^{\infty} \delta(x - i\pi). \end{aligned}$$

This function is known as a Dirac comb. It's an infinite series of evenly spaced delta functions as shown in the following graph.